A New Temporal Basis Function for the Time-Domain Integral Equation Method

Jin-Lin Hu, Chi H. Chan, *Senior Member, IEEE*, and Yuan Xu

***Abstract*—A new temporal basis function that has all-order continuous derivative has been constructed using a nonlinear optimization scheme. This new basis function provides a much more stable explicit marching-on-in-time (MOT) solution, based on the time-domain integral equation (TDIE) method, than what is presently available. Two examples are presented to illustrate the superior stability of the proposed temporal basis function.**

***Index Terms*—Nonlinear optimization, temporal basis function, time-domain integral equation method.**

I. Introduction

NUMERICALLY rigorous transient analyses are based either on the differential or integral equation approach. A time-domain integral equation (TDIE) method that requires only a surface discretization is sometimes preferred over the differential one using a volumetric discretization. Furthermore, TDIE implicitly imposes the radiation condition and there exists no grid dispersion. While TDIE has been around for over 30 years [1], its widespread use as an engineering tool has been deterred by three factors, namely, i) computational complexity of the algorithm, ii) availability of the required spatial-time domain Green's functions of inhomogeneous medium such as the layered media, and iii) stability of the marching-on-in-time (MOT) process. The first two factors have been addressed recently in [2]–[4], and further improvements are likely to be developed. In contrast, there is a continuous effort in searching for stable MOT schemes [5]–[8]. Each of these schemes in [5]–[8] pushes the late-time instability further down in time but could not eliminate it completely unless an implicit scheme, such as the one proposed in [9], which requires solving a large matrix equation, is employed. In [8], there is evidence that TDIE that employs a temporal basis function with a continuous derivative would provide a more stable MOT scheme. In this paper, we introduce a new temporal basis function that has continuous all-order derivative which leads to a much more stable explicit MOT solution than what are presently available.

Manuscript received May 10, 2001; revised September 4, 2001. This work was supported by the Hong Kong Research Grant Council under Grant 9 040 623. The review of this letter was arranged by Associate Editor Dr. Arvind Sharma.

J.-L. Hu is with the Wireless Communications Research Center, City University of Hong Kong, Kowloon, Hong Kong. He is also with the Department of Applied Mathematics, Northwestern Polytechnical University, Xian, Shaanxi, China.

C. H. Chan is with the Wireless Communications Research Center, City University of Hong Kong, Kowloon, Hong Kong.

Y. Xu is with the Millimeter Wave Technique Laboratory, Nanjing University of Science & Technology, Nanjing, China.

Publisher Item Identifier S 1531-1309(01)10794-4.

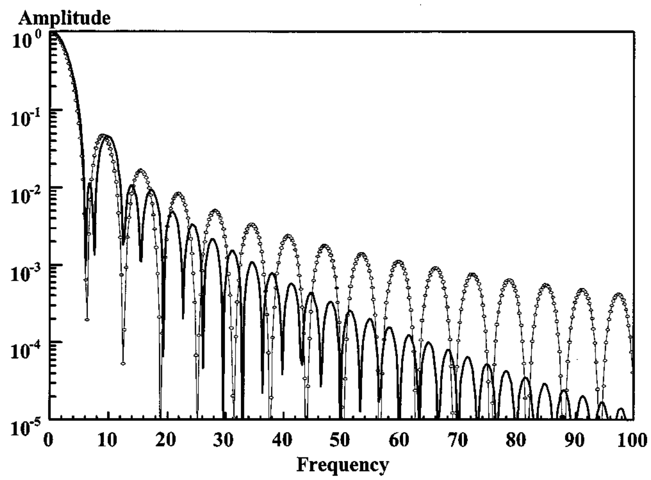

Fig. 1. Comparison of the spectral content of the triangular temporal basis function and our new basis function (real line: spectral content of our new basis function; dotted line: spectral content of triangular basis function).

II. New Temporal Basis Function

To have a temporal basis function that has all-order continuous derivative, we choose the following form

$$T(t) = \begin{cases} \exp\left(-\frac{a_0 t^2}{(1-t^2)(1+a_1 t^2+a_2 t^4+\cdots+a_N t^{2N})}\right), & |t|<1 \\ 0, & \text{otherwise} \end{cases} \quad (1)$$

[shown in (1) at the top of the next page]. For simplicity, we choose $N = 1$ and the function spans from -1 to 1. The unknown coefficients a_0 and a_1 are determined using the optimal construction scheme proposed in [10]. In [10], a nonlinear optimization scheme is adopted to minimize an objective function which requires that $T(t-1) + T(t) + T(t-1) = 1$. Through this construction process, we obtain, from (1), the new temporal basis function as

$$T(t) = \begin{cases} \exp\left(-\frac{4.6487t^2}{(1-t^2)(1+5t^2)}\right), & |t|<1 \\ 0, & \text{otherwise.} \end{cases} \quad (2)$$

An explanation for the cause of instability is that if the temporal basis function has a rich high-frequency content, the spatial discretization may not be fine enough for these high frequencies. The accumulated error will lead to late-time instability. Fig. 1 shows the comparison of the spectral content of the triangular temporal basis function and the proposed basis function. It is clear that this new function has less high-frequency contents

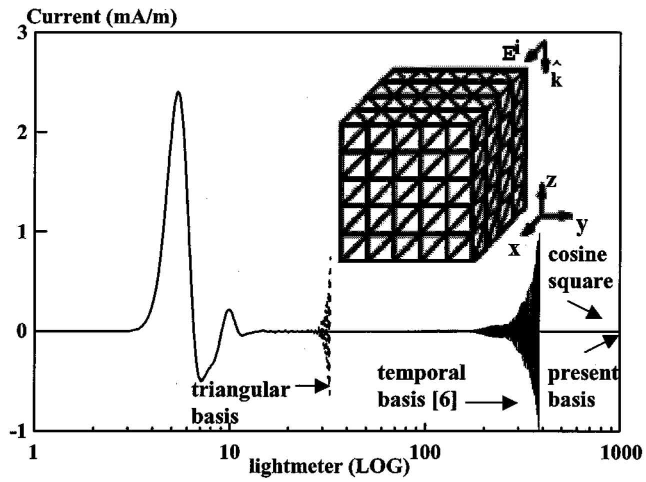

Fig. 2. Comparison of the current densities measured at the face center of the PEC cube using various temporal basis functions.

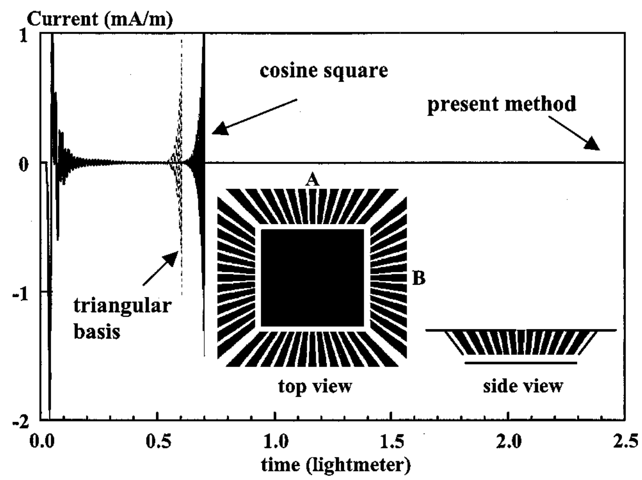

Fig. 3. Comparison of the current densities measured at Lead B using various temporal basis functions.

than that of the triangular one. It should be noted that, although not shown, the temporal basis function in (2) resembles that of the triangular basis function except that it has continuous derivatives at $t = -1, 0$ and 1. In addition, the condition that $T(t-1)+T(t)+T(t-1) = 1$ is only approximately satisfied.

III. Numerical Results

Two numerical examples are given to illustrate the superior stability of the temporal basis function given in (2). Both examples employ the RWG basis function [11] for spatial discretization and the averaging scheme proposed in [5]. The first example is a PEC cube with a plane wave incidence [6], [8]. The current density at one of the face centers computed by the TDIE method is shown in Fig. 2. It is shown that results using different temporal basis functions in [5], [6], [8] and the proposed one in (2), agree very well initially. However, as time elapses, the triangular basis function becomes unstable first, followed by the basis function proposed in [6]. The cosine square function, along with the proposed one in the present paper, continues to yield stable results for the duration shown in Fig. 2.

A more complicated example is shown in Fig. 3, in which a lead frame is excited by a current source at Lead A and the time-domain response is measured at Lead B. Once again, early-time results agree well among all different temporal basis functions. Instability occurs first for the triangular function followed by the cosine squared one. When replaced the temporal function by the proposed function in (2), no late-time instability is observed. It should be pointed out that the basis function in [6] requires a matrix inversion and therefore is not employed in this complicated example.

IV. Conclusion

We have constructed a temporal basis function through a nonlinear optimization scheme. This basis function has continuous all-order derivative. It also has much less high-frequency content than the commonly used triangular temporal basis function. It provides a superior stability for TDIE simulations as illustrated by the two examples given in the paper.

References

[1] C. L. Bennett and W. L. Weeks, “Transient scattering from conducting cylinders,” *IEEE Trans. Antennas Propagat.*, vol. AP–18, pp. 627–633, Sept. 1970.
[2] S. P. Walker, “Scattering analysis via time-domain integral equations: Methods to reduce the scaling of cost with frequency,” *IEEE Antennas Propagat. Mag.*, vol. 39, pp. 7–9, Feb. 1997.
[3] B. Shanker, A. A. Ergin, K. Aygun, and E. Michielssen, “Analysis of transient electromagnetic scattering phenomena using a two-level plane wave time-domain algorithm,” *IEEE Trans. Antennas Propagat.*, vol. 48, pp. 510–523, Apr. 2000.
[4] Y. Xu, D. G. Fang, M. Y. Xia, and C. H. Chan, “Speedy computation of the time-domain Green’s function for microstrip structures,” *Electron. Lett.*, vol. 36, pp. 1855–1857, Oct. 2000.
[5] D. A. Vechinski and S. M. Rao, “A stable procedure to calculate the transient scattering by conducting surface of arbitrary shaped,” *IEEE Trans. Antennas Propagat.*, vol. 40, pp. 661–665, June 1992.
[6] G. Manara, A. Monorchio, and R. Reggiannini, “A space-time discretization criterion for a stable time-marching solution of the electric field integral equation,” *IEEE Trans. Antennas Propagat.*, vol. 45, pp. 527–532, March 1997.
[7] P. J. Davies, “A stability analysis of a time marching scheme for the general surface electric field integral equation,” *Appl. Numer. Math.*, vol. 27, pp. 33–57, 1998.
[8] J.-L. Hu and C. H. Chan, “An improved temporal basis function for the time domain electric field integral equation method,” *Electron. Lett.*, vol. 35, pp. 883–885, May 1999.
[9] D. S. Weile, A. A. Ergin, B. Shanker, and E. Michielssen, “An accurate discretization scheme for the numerical solution of time domain integral equations,” in *Dig. 2000 IEEE Antennas Propagat. Int. Symp.*, Salt Lake City, UT, July 2000, pp. 741–744.
[10] J.-L. Hu and C. H. Chan, “On the construction of temporal basis function for the time-domain integral equation method,” *IEEE Trans. Antennas Propagat.*, to be published.
[11] S. M. Rao, D. R. Wilton, and A. W. Glisson, “Electromagnetic scattering by surfaces of arbitrary shape,” *IEEE Trans. Antennas Propagat.*, vol. AP-30, pp. 401–418, May 1982.